

## Chapter 111. Texas Essential Knowledge and Skills for Mathematics

### Subchapter B. Middle School

*Statutory Authority: The provisions of this Subchapter B issued under the Texas Education Code, §§7.102(c)(4), 28.002, and 28.008, unless otherwise noted.*

#### §111.21. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 6-8.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

*Source: The provisions of this §111.21 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 4479.*

#### §111.22. Mathematics, Grade 6.

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 6 are using ratios to describe direct proportional relationships involving number, geometry, measurement, probability, and adding and subtracting decimals and fractions.
- (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
- (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.

(b) Knowledge and skills.

- (1) Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms. The student is expected to:
  - (A) compare and order non-negative rational numbers;
  - (B) generate equivalent forms of rational numbers including whole numbers, fractions, and decimals;
  - (C) use integers to represent real-life situations;
  - (D) write prime factorizations using exponents;
  - (E) identify factors of a positive integer, common factors, and the greatest common factor of a set of positive integers; and
  - (F) identify multiples of a positive integer and common multiples and the least common multiple of a set of positive integers.

- (2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:
- (A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;
  - (B) use addition and subtraction to solve problems involving fractions and decimals;
  - (C) use multiplication and division of whole numbers to solve problems including situations involving equivalent ratios and rates;
  - (D) estimate and round to approximate reasonable results and to solve problems where exact answers are not required; and
  - (E) use order of operations to simplify whole number expressions (without exponents) in problem solving situations.
- (3) Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships. The student is expected to:
- (A) use ratios to describe proportional situations;
  - (B) represent ratios and percents with concrete models, fractions, and decimals; and
  - (C) use ratios to make predictions in proportional situations.
- (4) Patterns, relationships, and algebraic thinking. The student uses letters as variables in mathematical expressions to describe how one quantity changes when a related quantity changes. The student is expected to:
- (A) use tables and symbols to represent and describe proportional and other relationships such as those involving conversions, arithmetic sequences (with a constant rate of change), perimeter and area; and
  - (B) use tables of data to generate formulas representing relationships involving perimeter, area, volume of a rectangular prism, etc.
- (5) Patterns, relationships, and algebraic thinking. The student uses letters to represent an unknown in an equation. The student is expected to formulate equations from problem situations described by linear relationships.
- (6) Geometry and spatial reasoning. The student uses geometric vocabulary to describe angles, polygons, and circles. The student is expected to:
- (A) use angle measurements to classify angles as acute, obtuse, or right;
  - (B) identify relationships involving angles in triangles and quadrilaterals; and
  - (C) describe the relationship between radius, diameter, and circumference of a circle.
- (7) Geometry and spatial reasoning. The student uses coordinate geometry to identify location in two dimensions. The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.
- (8) Measurement. The student solves application problems involving estimation and measurement of length, area, time, temperature, volume, weight, and angles. The student is expected to:
- (A) estimate measurements (including circumference) and evaluate reasonableness of results;
  - (B) select and use appropriate units, tools, or formulas to measure and to solve problems involving length (including perimeter), area, time, temperature, volume, and weight;
  - (C) measure angles; and
  - (D) convert measures within the same measurement system (customary and metric) based on relationships between units.

- (9) Probability and statistics. The student uses experimental and theoretical probability to make predictions. The student is expected to:
- (A) construct sample spaces using lists and tree diagrams; and
  - (B) find the probabilities of a simple event and its complement and describe the relationship between the two.
- (10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to:
- (A) select and use an appropriate representation for presenting and displaying different graphical representations of the same data including line plot, line graph, bar graph, and stem and leaf plot;
  - (B) identify mean (using concrete objects and pictorial models), median, mode, and range of a set of data;
  - (C) sketch circle graphs to display data; and
  - (D) solve problems by collecting, organizing, displaying, and interpreting data.
- (11) Underlying processes and mathematical tools. The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
  - (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- (12) Underlying processes and mathematical tools. The student communicates about Grade 6 mathematics through informal and mathematical language, representations, and models. The student is expected to:
- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
  - (B) evaluate the effectiveness of different representations to communicate ideas.
- (13) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

*Source: The provisions of this §111.22 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1930.*

### **§111.23. Mathematics, Grade 7.**

- (a) Introduction.
- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 7 are using direct proportional relationships in number, geometry, measurement, and probability; applying

addition, subtraction, multiplication, and division of decimals, fractions, and integers; and using statistical measures to describe data.

- (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
  - (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.
- (b) Knowledge and skills.
- (1) Number, operation, and quantitative reasoning. The student represents and uses numbers in a variety of equivalent forms. The student is expected to:
    - (A) compare and order integers and positive rational numbers;
    - (B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator; and
    - (C) represent squares and square roots using geometric models.
  - (2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:
    - (A) represent multiplication and division situations involving fractions and decimals with models, including concrete objects, pictures, words, and numbers;
    - (B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;
    - (C) use models, such as concrete objects, pictorial models, and number lines, to add, subtract, multiply, and divide integers and connect the actions to algorithms;
    - (D) use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio;
    - (E) simplify numerical expressions involving order of operations and exponents;
    - (F) select and use appropriate operations to solve problems and justify the selections; and
    - (G) determine the reasonableness of a solution to a problem.
  - (3) Patterns, relationships, and algebraic thinking. The student solves problems involving direct proportional relationships. The student is expected to:
    - (A) estimate and find solutions to application problems involving percent; and
    - (B) estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

- (4) Patterns, relationships, and algebraic thinking. The student represents a relationship in numerical, geometric, verbal, and symbolic form. The student is expected to:
- (A) generate formulas involving unit conversions within the same system (customary and metric), perimeter, area, circumference, volume, and scaling;
  - (B) graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling; and
  - (C) use words and symbols to describe the relationship between the terms in an arithmetic sequence (with a constant rate of change) and their positions in the sequence.
- (5) Patterns, relationships, and algebraic thinking. The student uses equations to solve problems. The student is expected to:
- (A) use concrete and pictorial models to solve equations and use symbols to record the actions; and
  - (B) formulate problem situations when given a simple equation and formulate an equation when given a problem situation.
- (6) Geometry and spatial reasoning. The student compares and classifies two- and three-dimensional figures using geometric vocabulary and properties. The student is expected to:
- (A) use angle measurements to classify pairs of angles as complementary or supplementary;
  - (B) use properties to classify triangles and quadrilaterals;
  - (C) use properties to classify three-dimensional figures, including pyramids, cones, prisms, and cylinders; and
  - (D) use critical attributes to define similarity.
- (7) Geometry and spatial reasoning. The student uses coordinate geometry to describe location on a plane. The student is expected to:
- (A) locate and name points on a coordinate plane using ordered pairs of integers; and
  - (B) graph reflections across the horizontal or vertical axis and graph translations on a coordinate plane.
- (8) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:
- (A) sketch three-dimensional figures when given the top, side, and front views;
  - (B) make a net (two-dimensional model) of the surface area of a three-dimensional figure; and
  - (C) use geometric concepts and properties to solve problems in fields such as art and architecture.
- (9) Measurement. The student solves application problems involving estimation and measurement. The student is expected to:
- (A) estimate measurements and solve application problems involving length (including perimeter and circumference) and area of polygons and other shapes;
  - (B) connect models for volume of prisms (triangular and rectangular) and cylinders to formulas of prisms (triangular and rectangular) and cylinders; and
  - (C) estimate measurements and solve application problems involving volume of prisms (rectangular and triangular) and cylinders.

- (10) Probability and statistics. The student recognizes that a physical or mathematical model (including geometric) can be used to describe the experimental and theoretical probability of real-life events. The student is expected to:
- (A) construct sample spaces for simple or composite experiments; and
  - (B) find the probability of independent events.
- (11) Probability and statistics. The student understands that the way a set of data is displayed influences its interpretation. The student is expected to:
- (A) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection; and
  - (B) make inferences and convincing arguments based on an analysis of given or collected data.
- (12) Probability and statistics. The student uses measures of central tendency and variability to describe a set of data. The student is expected to:
- (A) describe a set of data using mean, median, mode, and range; and
  - (B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.
- (13) Underlying processes and mathematical tools. The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
  - (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
- (14) Underlying processes and mathematical tools. The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models. The student is expected to:
- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
  - (B) evaluate the effectiveness of different representations to communicate ideas.
- (15) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

*Source: The provisions of this §111.23 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1930; amended to be effective February 22, 2009, 34 TexReg 1056.*

**§111.24. Mathematics, Grade 8.**

- (a) Introduction.
- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent both proportional and non-proportional linear relationships and using probability to describe data and make predictions.
  - (2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships. Students use geometric properties and relationships, as well as spatial reasoning, to model and analyze situations and solve problems. Students communicate information about geometric figures or situations by quantifying attributes, generalize procedures from measurement experiences, and use the procedures to solve problems. Students use appropriate statistics, representations of data, reasoning, and concepts of probability to draw conclusions, evaluate arguments, and make recommendations.
  - (3) Problem solving in meaningful contexts, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 6-8, students use these processes together with graphing technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve problems as they do mathematics.
- (b) Knowledge and skills.
- (1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:
    - (A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
    - (B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;
    - (C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (such as  $\pi$ ,  $\sqrt{2}$ );
    - (D) express numbers in scientific notation, including negative exponents, in appropriate problem situations; and
    - (E) compare and order real numbers with a calculator.
  - (2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:
    - (A) select appropriate operations to solve problems involving rational numbers and justify the selections;
    - (B) use appropriate operations to solve problems involving rational numbers in problem situations;
    - (C) evaluate a solution for reasonableness; and
    - (D) use multiplication by a given constant factor (including unit rate) to represent and solve problems involving proportional relationships including conversions between measurement systems.

- (3) Patterns, relationships, and algebraic thinking. The student identifies proportional or non-proportional linear relationships in problem situations and solves problems. The student is expected to:
  - (A) compare and contrast proportional and non-proportional linear relationships; and
  - (B) estimate and find solutions to application problems involving percents and other proportional relationships such as similarity and rates.
- (4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation of data given another representation of data (such as a table, graph, equation, or verbal description).
- (5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:
  - (A) predict, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and
  - (B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).
- (6) Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. The student is expected to:
  - (A) generate similar figures using dilations including enlargements and reductions; and
  - (B) graph dilations, reflections, and translations on a coordinate plane.
- (7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:
  - (A) draw three-dimensional figures from different perspectives;
  - (B) use geometric concepts and properties to solve problems in fields such as art and architecture;
  - (C) use pictures or models to demonstrate the Pythagorean Theorem; and
  - (D) locate and name points on a coordinate plane using ordered pairs of rational numbers.
- (8) Measurement. The student uses procedures to determine measures of three-dimensional figures. The student is expected to:
  - (A) find lateral and total surface area of prisms, pyramids, and cylinders using concrete models and nets (two-dimensional models);
  - (B) connect models of prisms, cylinders, pyramids, spheres, and cones to formulas for volume of these objects; and
  - (C) estimate measurements and use formulas to solve application problems involving lateral and total surface area and volume.
- (9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:
  - (A) use the Pythagorean Theorem to solve real-life problems; and
  - (B) use proportional relationships in similar two-dimensional figures or similar three-dimensional figures to find missing measurements.

- (10) Measurement. The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:
- (A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and
  - (B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.
- (11) Probability and statistics. The student applies concepts of theoretical and experimental probability to make predictions. The student is expected to:
- (A) find the probabilities of dependent and independent events;
  - (B) use theoretical probabilities and experimental results to make predictions and decisions; and
  - (C) select and use different models to simulate an event.
- (12) Probability and statistics. The student uses statistical procedures to describe data. The student is expected to:
- (A) use variability (range, including interquartile range (IQR)) and select the appropriate measure of central tendency to describe a set of data and justify the choice for a particular situation;
  - (B) draw conclusions and make predictions by analyzing trends in scatterplots; and
  - (C) select and use an appropriate representation for presenting and displaying relationships among collected data, including line plots, line graphs, stem and leaf plots, circle graphs, bar graphs, box and whisker plots, histograms, and Venn diagrams, with and without the use of technology.
- (13) Probability and statistics. The student evaluates predictions and conclusions based on statistical data. The student is expected to:
- (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and
  - (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.
- (14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
  - (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
  - (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
  - (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

- (15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:
- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
  - (B) evaluate the effectiveness of different representations to communicate ideas.
- (16) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

*Source: The provisions of this §111.24 adopted to be effective September 1, 1998, 22 TexReg 7623; amended to be effective August 1, 2006, 30 TexReg 1930; amended to be effective February 22, 2009, 34 TexReg 1056.*